December 4th, 2021 Professor Ebou Janha Company QCT OF USA

How to determine the coordinates of the center of a circle, its radius, slope and its tangent line in reference to a particular point on the circle and the slope of the tangent line.

$$X^2 - 4X + y^2 + 2y - 31 = 0$$

Step 1

Rearrange the equation to $X^2 - 4X + y^2 + 2y = 31$

Step 2

Complete the square. To complete the square, partition the equation into two segments.

That produces

$$\underbrace{X^2 - 4X + \_\_\_}_{\text{segment 1}} + \underbrace{Y^2 + 2Y + \_\_\_}_{\text{segment 2}} = 31$$

Step 3

Now divide the second number on each segment by 2, square the answer and add it next to the number.

That becomes $\left(\frac{4}{2}\right)^2 = 4$ $\left(\frac{2}{2}\right)^2 = `1$

Continue to page two for Step 4

Step 4

Add both numbers to the right of the equation next to 31

That becomes

$$X^2 - 4X + 4 + Y^2 + 2Y + 1 = 31 + 4 + 1$$

Step 5

Now simplify the equation

That becomes

$$X^2 - 4X + 4 + Y^2 + 2Y + 1 = 36$$

Step 6

Now bring the format to determine the center and the radius. To do that, you bring the **X** add the sign next to it **+ or –** then bring the number you squared in this case **2** and square it. Do that on both segments.

That becomes

$$(X-2)^2 + (Y+1)^2 = 36$$ **where 36 is** r^2 **or radius**^{2}

So the coordinates **(X, Y)** of the center of the circle simply becomes the reverse of the signs **(+ -)** of the values of the **X** and **Y** and the radius is the square root of **36** which is **6**

That becomes

Central is (2, -1) the radius of the circle is 6

Continue to page 3 for step 7

Step 7

Graph the circle with the central coordinates **(2, -1)** with a radius of **6** and find the slope of the radius.

Slope of the radius

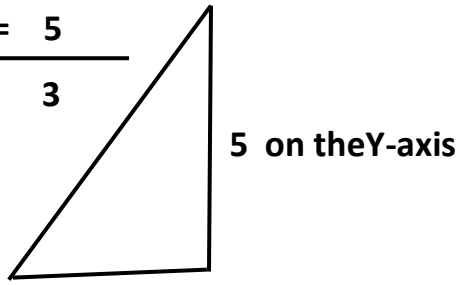
$$\text{Slope} = \frac{Y2 - Y1}{X2 - X1}$$

$$= \frac{4 - -1}{5 - 2} \qquad = \frac{5}{3}$$

Another method for slope

The height of the triangle formed by the line divide by the width

$$= \frac{5}{3}$$



Continue to page 4 for step 8

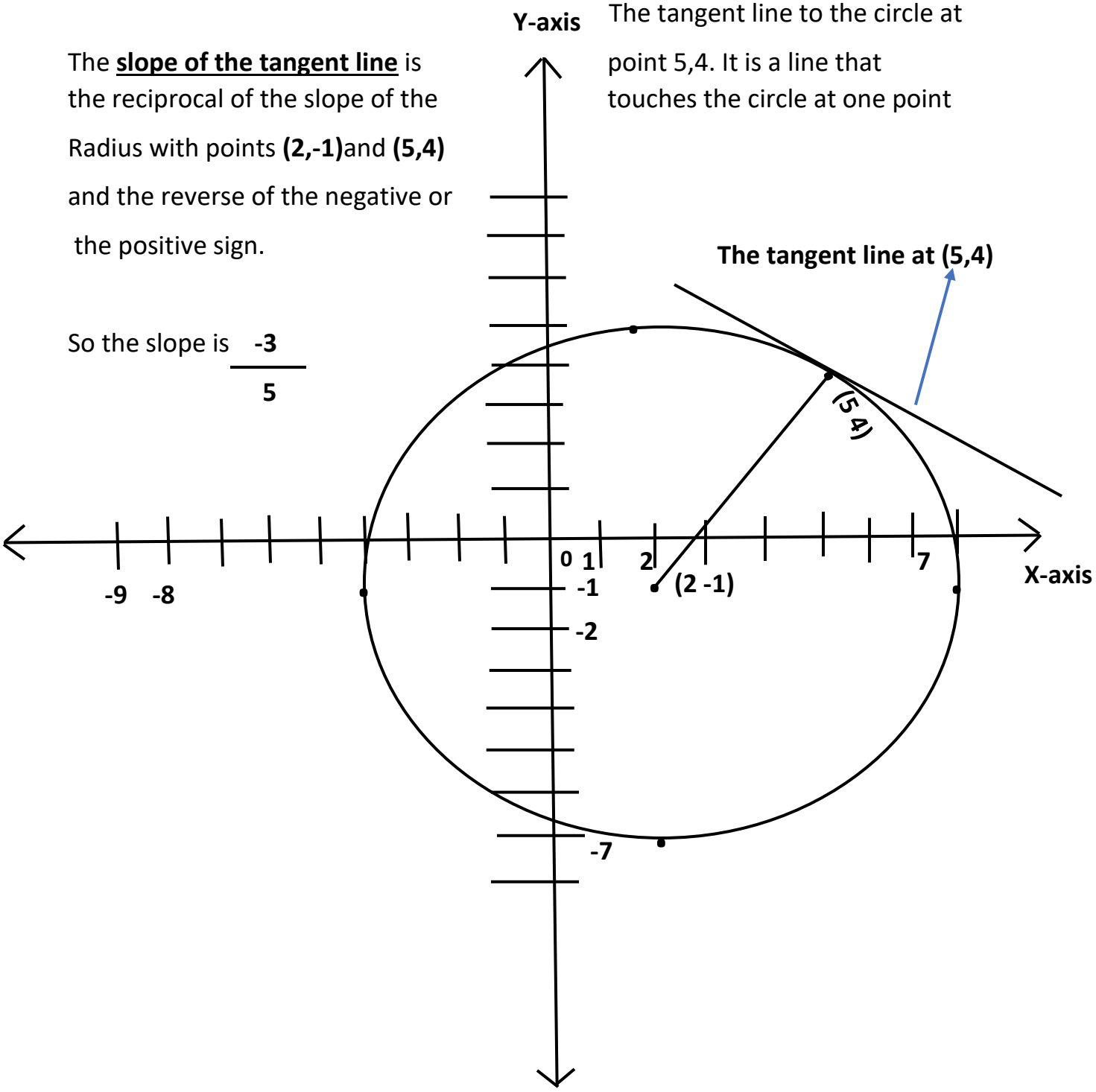
Step 8

Now find the tangent line of the circle at point **(5,4)** and its slope.

The **slope of the tangent line** is the reciprocal of the slope of the Radius with points **(2,-1)**and **(5,4)** and the reverse of the negative or the positive sign.

So the slope is $\frac{-3}{5}$

The tangent line to the circle at point 5,4. It is a line that touches the circle at one point



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Step 9

Find the equation of the tangent line. To find the equation, we apply the formulae below.

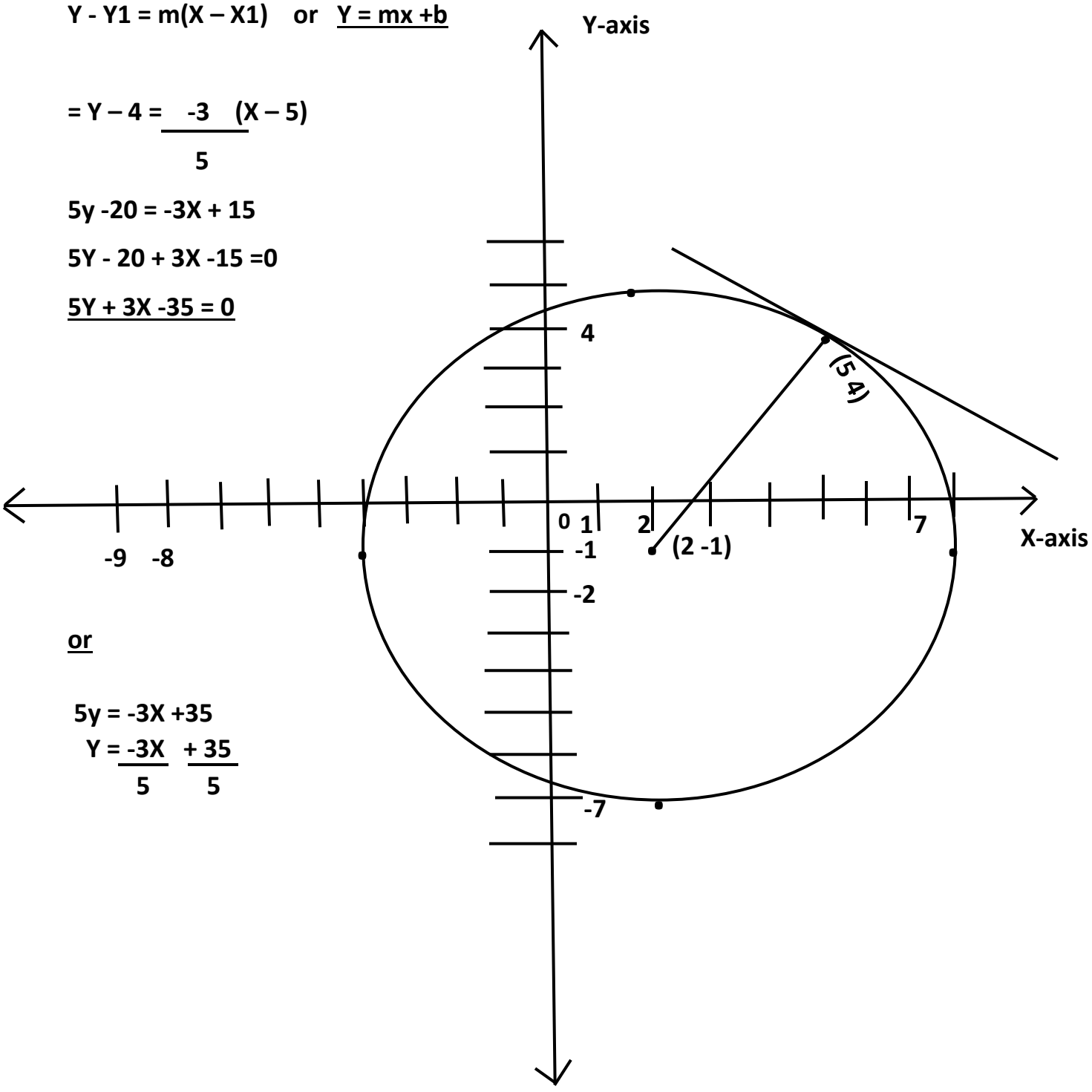
Y - Y1 = m(X – X1) or Y = mx +b

= Y – 4 = -3 (X – 5)
5

5y -20 = -3X + 15

5Y - 20 + 3X -15 =0

5Y + 3X -35 = 0



or

5y = -3X +35

Y = -3X + 35
5 5